Meson Mixing in Pion Superfluid

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We investigate the meson mixing and meson coupling constants in pion superfluid for σ and π in the frame of two flavor NJL model. The mixing strength develops fast with increasing isospin chemical potential, and the coupling constants in the normal phase and in the superfluidity phase behavior very differently.

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I. INTRODUCTION

Recently, the study on the phase diagram of Quantum Chromodynamics (QCD) is extended from finite temperature and baryon chemical potential to finite isospin chemical potential[1, 2, 3, 4, 5, 6, 7]. The physical motivation to study the pion superfluid formed at high isospin density is related to the investigation of neutron stars, isospin asymmetric nuclear matter and heavy ion collisions at intermediate energy.

The familiar meson properties are significantly changed in the pion superfluidity phase. In the normal phase which is controlled by spontaneous chiral symmetry breaking and its restoration, the scalar and pseudoscalar meson polarizations of a two flavor quark system are σ and π. However, they are no longer the eigen modes of the Hamiltonian of the system in the pion superfluid, and the new eigen modes are the linear combination of them[8]. It is the combination that guarantees the Goldstone meson corresponding to the spontaneous isospin symmetry breaking. To investigate the dynamic processes which may be considered as signatures of pion superfluid, one needs not only the meson masses but also other in-medium meson preperties in the superfluid. In this Letter, we will study the meson mixing and meson coupling constants for σ and π at finite isospin density in the frame of an effective QCD model.

The perturbation theory of QCD can well describe the properties of the new phases in the phase diagram in high temperature and high density regions, but the study on the phase structure at moderate temperature and density depends on lattice QCD calculation and effective models with QCD symmetries. One of the models that enables us to see directly how the dynamic mechaisms of chiral symmetry breaking and restoration operate is the Nambu-Jona-Lasinio model (NJL)[9] applied to quarks[10]. The chiral phase transition line[11] in the temperature and baryon chemical potential plane calculated in the model is very close to the one obtained with lattice QCD. Recently, this model was used to investigate the color superconductivity[12, 13, 14, 15] at moderate baryon density and pion superfluidity[8, 16, 17, 18, 19, 20] at finite isospin density. We will study the meson properties in the pion superfluid in the frame of this model.

The Letter is organized as follows. In Section II we review the NJL model in mean field approximation for quarks and random phase approximation (RPA) for mesons in the normal phase at finite isospin chemical potential. In Section III we focus on the meson mixing and meson couplings to quarks in the pion superfluid. We summarize in Section IV.

II. MESONS IN NORMAL PHASE

The meson properties in the normal phase are characterized by the chiral dynamics and explicit isospin symmetry breaking. In this section we review the quark propagator in mean field approximation and meson polarizations in RPA in the NJL model at finite temperature and baryon and isospin chemical potentials. The Lagrangian density of the two flavor NJL model at quark level is defined as[10]

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma_0\right)\psi + G\Big[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau_i\psi\right)^2\Big] \quad (1)$$

with scalar and pseudoscalar interactions corresponding to σ, π_+, π_- and π_0 excitations, where m_0 is the current quark mass, G is the four-quark coupling constant with dimension $(\mathrm{GeV})^{-2}$, τ_i $(i = 1, 2, 3)$ are the Pauli matrices in flavor space, and $\mu = diag\left(\mu_u, \mu_d\right) = diag\left(\mu_B/3 + \mu_I/2, \mu_B/3 - \mu_I/2\right)$ is the quark chemical potential matrix with μ_u and μ_d being the u- and d-quark chemical potentials and μ_B and μ_I being the baryon and isospin chemical potentials.

At zero isospin chemical potential, the Lagrangian density has the symmetry $U_B(1)\bigotimes SU_I(2)\bigotimes SU_A(2)$ corresponding to baryon number gauge symmetry, isospin symmetry and chiral symmetry. At nonzero isospin chemical potential, the isospin symmetry $SU_I(2)$ breaks explicitly to $U_I(1)$ global symmetry, and the chiral symmetry $SU_A(2)$ breaks explicitly to $U_A(1)$ global symmetry. Therefore, the chiral symmetry restoration at finite isospin chemical potential means only the degeneracy of σ and π_0 mesons only, and the charged π_+ and π_- behavior differently.

Introducing the chiral and pion condensates

$$\begin{aligned}\sigma &= \langle\bar\psi\psi\rangle = \sigma_u+\sigma_d,\\ \sigma_u &= \langle\bar u u\rangle, \quad \sigma_d = \langle\bar d d\rangle,\\ \pi &= \sqrt{2}\langle\bar\psi i\gamma_5\tau_+\psi\rangle = \sqrt{2}\langle\bar\psi i\gamma_5\tau_-\psi\rangle,\end{aligned} \tag{2}$$

where $\tau_\pm$ are defined as $\tau_\pm = (\tau_1 \pm i\tau_2)/\sqrt{2}$ and we have taken all the condensates to be real, the quark propagator in mean field approximation can be expressed as a matrix in the flavor space[8]

$$\mathcal{S}(p) = \begin{pmatrix} \mathcal{S}_{uu}(p) & \mathcal{S}_{ud}(p)\\ \mathcal{S}_{du}(p) & \mathcal{S}_{dd}(p)\end{pmatrix} \tag{3}$$

with the four elements[8]

$$\begin{aligned}\mathcal{S}_{uu} &= \frac{(p_0+E+\mu_d)\,\Lambda_+\gamma_0}{(p_0-E_-^-)(p_0+E_+^-)} + \frac{(p_0-E+\mu_d)\,\Lambda_-\gamma_0}{(p_0-E_-^+)(p_0+E_+^+)},\\ \mathcal{S}_{dd} &= \frac{(p_0-E+\mu_u)\,\Lambda_-\gamma_0}{(p_0-E_-^-)(p_0+E_+^-)} + \frac{(p_0+E+\mu_u)\,\Lambda_+\gamma_0}{(p_0-E_-^+)(p_0+E_+^+)},\\ \mathcal{S}_{ud} &= \frac{2iG\pi\Lambda_+\gamma_5}{(p_0-E_-^-)(p_0+E_+^-)} + \frac{2iG\pi\Lambda_-\gamma_5}{(p_0-E_-^+)(p_0+E_+^+)},\\ \mathcal{S}_{du} &= \frac{2iG\pi\Lambda_-\gamma_5}{(p_0-E_-^-)(p_0+E_+^-)} + \frac{2iG\pi\Lambda_+\gamma_5}{(p_0-E_-^+)(p_0+E_+^+)},\end{aligned} \tag{4}$$

where $E_\mp^\pm(p)$ are energies of the four quasi-particles $E_\mp^\pm = E^\pm \mp \mu_B/3$ with $E^\pm = \sqrt{(E\pm\mu_I/2)^2+4G^2\pi^2}$, $E = \sqrt{|\mathbf{p}|^2+M_q^2}$ and the effective quark mass $M_q = m_0 - 2G\sigma$ controlled by the chiral condensate, and $\Lambda_\pm(\mathbf{p})$ are the energy projectors $\Lambda_\pm = (1\pm\gamma_0(\gamma\cdot\mathbf{p}+M_q)/E)/2$. The quark propagator (3) is the background of our calculations for quarks and mesons. From the definitions of the chiral and pion condensates (2), it is easy to express them in terms of the matrix elements of the quark propagator,

$$\begin{aligned}\sigma_u &= -\sum_p Tr\left[i\mathcal{S}_{uu}(p)\right],\\ \sigma_d &= -\sum_p Tr\left[i\mathcal{S}_{dd}(p)\right],\\ \pi &= \sum_p Tr\left[(\mathcal{S}_{ud}(p)+\mathcal{S}_{du}(p))\,\gamma_5\right],\end{aligned} \tag{5}$$

where the trace $Tr = Tr_C Tr_D$ is taken in color and Dirac spaces, and the four momentum integration is defined as $\sum_p = iT\sum_n\int d^3\mathbf{p}/(2\pi)^3$ with $p_0 = i\omega_n = i(2n+1)\pi T$ at finite temperature T. It is necessary to note that for $\mu_B = 0$ or $\mu_I = 0$, there is always $\sigma_u = \sigma_d$, since in this case the chemical potential difference between $\bar u$ and u is the same as the difference between $\bar d$ and d. The coupled set of gap equations (5) determines self-consistently the three condensates as functions of T, μ_B and μ_I.

In the NJL model, the meson modes are regarded as quantum fluctuations above the mean field. The two quark scattering via meson exchange can be effectively expressed at quark level in terms of quark bubble summation in RPA. The quark bubbles, namely the meson polarization functions are defined as[10, 11]

$$\Pi_{mn}(k) = i\sum_p Tr\left(\Gamma_m^*\mathcal{S}(p+k)\Gamma_n\mathcal{S}(p)\right) \tag{6}$$

with the trace $Tr = Tr_C Tr_F Tr_D$ taken in color, flavor and Dirac spaces and the meson vertexes

$$\Gamma_m = \begin{cases} 1 & m=\sigma\\ i\tau_+\gamma_5 & m=\pi_+\\ i\tau_-\gamma_5 & m=\pi_-\\ i\tau_3\gamma_5 & m=\pi_0\ ,\end{cases} \qquad \Gamma_m^* = \begin{cases} 1 & m=\sigma\\ i\tau_-\gamma_5 & m=\pi_+\\ i\tau_+\gamma_5 & m=\pi_-\\ i\tau_3\gamma_5 & m=\pi_0\ .\end{cases} \tag{7}$$

The explicit T, μ_B and μ_I dependence of the meson polarization functions (6) used in the following discussion can be found in Appendix B of [8].

At $\mu_I \le \mu_I^c$ where μ_I^c is the critical isospin chemical potential for pion condensation, the system is in the normal phase with diagonal quark propagator, and the bubble summation in the construction of the effective interaction in RPA selects its specific isospin channel by choosing at each stage the same proper polarization[10]. Therefore, the meson masses $M_m (m = \sigma, \pi_+, \pi_-, \pi_0)$ which are determined by the poles of the meson propagators at $k_0 = M_m$ and $\mathbf{k} = 0$ are related to their own polarization functions only[10],

$$1 - 2G\Pi_{mm}(k_0)|_{k_0=M_m} = 0. \tag{8}$$

From the comparison of these mass equations with the gap equation for the pion condensate π at $T = \mu_B = 0$ but finite μ_I, the critical isospin chemical potential is exactly the pion mass in the vacuum[2, 8, 19], $\mu_I^c = m_\pi$, and the μ_I dependence of the meson masses is simple[8], $M_\sigma(\mu_I) = m_\sigma$, $M_{\pi_0}(\mu_I) = m_\pi$ and $M_{\pi_\pm}(\mu_I) = m_\pi \mp \mu_I$, where m_σ and m_π are the σ and π masses at $T = \mu_B = \mu_I = 0$. The isospin neutral mesons keep their vacuum masses, and the isospin charged mesons change their masses linearly in the isospin chemical potential. These relations hold until the pion condensation starts.

The meson couplings to quarks, $g_{mq\bar q}$, are related to the residues at the corresponding poles of the meson propagators[10],

$$g_{mq\bar q}^2 = \left[\partial\Pi_{mm}(k_0)/\partial k_0^2\right]^{-1}|_{k_0=M_m}. \tag{9}$$

III. MESONS IN PION SUPERFLUID

In the phase with finite pion condensate due to spontaneous isospin symmetry breaking at $\mu_I > \mu_I^c$, the quark propagator contains off-diagonal elements, we must consider all possible channels in the bubble summation in RPA[10]. While there is no mixing between π_0 and other mesons[8], $\Pi_{\pi_0\sigma}(k) = \Pi_{\pi_0\pi_+}(k) = \Pi_{\pi_0\pi_-}(k) = 0$, the other three mesons are coupled together, and the effective interaction via exchanging these mesons in RPA becomes a summation in the meson space,

$$U(k) = \Gamma_m^* \mathcal{M}_{mn}(k)\Gamma_n, \quad m,n = \sigma,\ \pi_+,\ \pi_- \tag{10}$$

with the meson matrix $\mathcal{M}(k)$ defined by

$$\mathcal{M}(k) = \frac{2G}{1-2G\Pi(k)} = \frac{2G}{D(k)}\overline{\mathcal{M}}(k), \tag{11}$$

where $1-2G\Pi(k)$ is the polarization matrix[8]

$$\begin{aligned} & 1-2G\Pi \\ = & \begin{pmatrix} 1-2G\Pi_{\sigma\sigma} & -2G\Pi_{\sigma\pi_+} & -2G\Pi_{\sigma\pi_-} \\ -2G\Pi_{\pi_+\sigma} & 1-2G\Pi_{\pi_+\pi_+} & -2G\Pi_{\pi_+\pi_-} \\ -2G\Pi_{\pi_-\sigma} & -2G\Pi_{\pi_-\pi_+} & 1-2G\Pi_{\pi_-\pi_-} \end{pmatrix}, \end{aligned} \tag{12}$$

$D(k)$ is its determinant,

$$D(k) = \det\left(1-2G\Pi(k)\right), \tag{13}$$

and $\overline{\mathcal{M}}(k)$ is defined as $\overline{\mathcal{M}}(k) = D(k)/(1-2G\Pi(k))$. In this case, σ, π_+ and π_- are no longer the eigen modes of the Hamiltonian of the system, the new eigen modes are linear combinations of them. In the following we call these new eigen modes in the superfluidity phase as $\overline{\sigma}, \overline{\pi}_+$ and $\overline{\pi}_-$.

The π_0 mass and coupling constant are still controlled by its own polarization function,

$$\begin{aligned} & 1-2G\Pi_{\pi_0\pi_0}(k_0)|_{k_0=M_{\pi_0}} = 0, \\ & g^2_{\pi_0 q\bar{q}} = \left[\partial\Pi_{\pi_0\pi_0}(k_0)/\partial k_0^2\right]^{-1}|_{k_0=M_{\pi_0}}, \end{aligned} \tag{14}$$

since it is independent of the other collective modes. At $T=\mu_B=0$, the π_0 mass is exactly equal to the isospin chemical potential[8], $M_{\pi_0}(\mu_I) = \mu_I$. The masses of the new eigen modes are determined by the pole of the effective interaction,

$$D(k_0)|_{k_0=M_\theta} = 0, \quad \theta = \overline{\sigma}, \overline{\pi}_+, \overline{\pi}_-. \tag{15}$$

It can be proven that there is always a zero solution which guarantees the Goldstone mode, $M_{\overline{\pi}_+} = 0$, corresponding to the spontaneous isospin symmetry breaking[8].

In order to derive the coupling constants for the new modes, we first expand the effective interaction U around the meson mass $k_0^2 = M_\theta^2$ at $\mathbf{k} = 0$,

$$U(k_0) \simeq \frac{2G}{(dD(k_0)/dk_0^2)|_{k_0=M_\theta}} \frac{\Gamma_m^* \overline{\mathcal{M}}_{mn}(M_\theta)\Gamma_n}{k_0^2 - M_\theta^2}, \tag{16}$$

and then try to make transformation from the coupled meson space σ, π_+, π_- to the independent meson space $\overline{\sigma}, \overline{\pi}_+, \overline{\pi}_-$. With the help of the pole equation (15) for the mass M_θ, we can derive the relations between the diagonal and off-diagonal elements of the matrix $\overline{\mathcal{M}}$,

$$\begin{aligned} & \overline{\mathcal{M}}_{mm}\overline{\mathcal{M}}_{nn} - \overline{\mathcal{M}}^2_{mn} = (1-2G\Pi_{ll})D = 0, \\ & l,m,n = \sigma, \pi_+, \pi_-, \quad l \neq m \neq n, \end{aligned} \tag{17}$$

where all the quantities are evaluated at $k_0 = M_\theta$. Taking into account the symmetric property of $\overline{\mathcal{M}}$,

$$\overline{\mathcal{M}}_{mn}(M_\theta) = \overline{\mathcal{M}}_{nm}(M_\theta), \tag{18}$$

the effective interaction U in the θ-meson channel can be written as

$$U(k_0) \simeq \frac{2G\overline{M}(M_\theta)}{(dD(k_0)/dk_0^2)|_{k_0=M_\theta}} \frac{\Gamma_\theta^*\Gamma_\theta}{k_0^2 - M_\theta^2} \tag{19}$$

in terms of the new meson vertex

$$\begin{aligned} \Gamma_\theta &= \sum_m \sqrt{\overline{\mathcal{M}}_{mm}(M_\theta)}\ \Gamma_m \Big/ \sqrt{\overline{M}(M_\theta)}, \\ \Gamma_\theta^* &= \sum_m \sqrt{\overline{\mathcal{M}}_{mm}(M_\theta)}\ \Gamma_m^* \Big/ \sqrt{\overline{M}(M_\theta)}, \end{aligned} \tag{20}$$

where $\overline{M}(M_\theta)$ is defined as $\overline{M}(M_\theta) = \sum_m \overline{\mathcal{M}}_{mm}(M_\theta)$.

From the definition of the meson coupling constant as the residue of the effective interaction at the pole, we can extract the θ-meson coupling constant $g_{\theta q\bar{q}}$ from (19),

$$g^2_{\theta q\bar{q}} = \frac{2G\overline{M}(M_\theta)}{(dD(k_0)/dk_0^2)|_{k_0=M_\theta}}. \tag{21}$$

To explicitly describe the meson mixing in the pion superfluidity phase, we can introduce mixing angles between two mesons, for instance, the angels α between π_+ and σ, β between π_- and σ and γ between π_+ and π_- in the $\overline{\sigma}$-meson channel,

$$\begin{aligned} \tan\alpha &= \sqrt{\overline{\mathcal{M}}_{\pi_+\pi_+}(M_{\overline{\sigma}})}\Big/\sqrt{\overline{\mathcal{M}}_{\sigma\sigma}(M_{\overline{\sigma}})}, \\ \tan\beta &= \sqrt{\overline{\mathcal{M}}_{\pi_-\pi_-}(M_{\overline{\sigma}})}\Big/\sqrt{\overline{\mathcal{M}}_{\sigma\sigma}(M_{\overline{\sigma}})}, \\ \tan\gamma &= \sqrt{\overline{\mathcal{M}}_{\pi_+\pi_+}(M_{\overline{\sigma}})}\Big/\sqrt{\overline{\mathcal{M}}_{\pi_-\pi_-}(M_{\overline{\sigma}})} \\ &= \tan\alpha/\tan\beta. \end{aligned} \tag{22}$$

It is easy to see that only α and β are independent. The mixing angels in the $\overline{\pi}_+$- and $\overline{\pi}_-$-meson channels can be defined in the similar way.

Since the NJL model is non-renormalizable, we should employ a hard three momentum cutoff Λ to regularize the gap equations for quarks and pole equations for mesons. In the following numerical calculations, we take the current quark mass $m_0 = 5$ MeV, the coupling constant $G = 4.93$ GeV^{-2} and the cutoff $\Lambda = 653$ MeV[11]. This group of parameters ensures the pion mass $m_\pi = 134$ MeV and the pion decay constant $f_\pi = 93$ MeV in the vacuum.

The mixing angles α, β and γ in the $\overline{\sigma}$-meson channel are shown in Fig.1 as functions of μ_I at $T = \mu_B = 0$. In the normal phase with $\mu_I \le \mu_I^c = m_\pi$, σ and π are the collective excitation modes of the system, and there is no any mixing among them. In the pion superfluidity phase with $\mu_I > \mu_I^c$, $\overline{\sigma}, \overline{\pi}_+$ and $\overline{\pi}_-$ become the new eigen modes of the system. In the $\overline{\sigma}$-meson channel, α and β

indicate the $\pi_+ - \sigma$ and $\pi_- - \sigma$ mixing strengths, and γ reflects the relative strength between them. While in the very beginning of the superfluid any mixing is weak, it develops fast. For $\mu_I > 200$ MeV which corresponds to $\alpha = \pi/4$, the mixing is already so strong that the contribution from π_+ to $\overline{\sigma}$ is larger than that from σ. Similarly, the π_--component in $\overline{\sigma}$ becomes more important than the σ-component itself even for $\mu_I > 150$ MeV. Therefore, at not very high isospin chemical potential the π_-- and π_+-components start to dominate the $\overline{\sigma}$ mesons. While the $\pi_- - \sigma$ mixing is always stronger than the $\pi_+ - \sigma$ mixing, namely $\beta > \alpha$, the relative strength shown by γ decreases with increasing isospin density. From our numerical calculation, the mixing angles in the $\overline{\pi}_-$-meson channel behavior similarly. However, the case is significantly changed for the Goldstone mode. The vertex for $\overline{\pi}_+$ can be greatly simplified as

$$\Gamma_{\overline{\pi}_+} = \left(\Gamma_{\pi_+} - \Gamma_{\pi_-}\right)/\sqrt{2} \tag{23}$$

at any temperature and baryon and isospin chemical potentials. The Goldstone mode contains only π_+- and π_--components, and the fractions for the two components are exactly the same.

The meson couplings to quarks are shown in Fig.2 as functions of μ_I at $T = \mu_B = 0$. In the normal phase, the coupling constants for σ and π are calculated through their own polarization functions. At $T = \mu_B = 0$, Π_{mm} depends only on the quark mass M_q, meson mass M_m and isospin chemical potential μ_I. Since M_q is a constant in the normal phase, and for the isospin neutral mesons σ and π_0 their masses are also constants in the region with only explicit isospin breaking, the coupling constants $g_{\sigma q\bar{q}}$ and $g_{\pi_0 q\bar{q}}$ are μ_I independent at $\mu_I < \mu_I^c$. The energy condition for a meson to decay into two quarks is that its mass should be larger than two times the effective quark mass. In the superfluidity phase the meson masses $M_{\overline{\sigma}}, M_{\overline{\pi}_+}, M_{\overline{\pi}_-}$ and M_{π_0} are calculated through the pole equations (14) and (15), see the Fig.15 of [8]. The heaviest mode is $\overline{\sigma}$, its mass is beyond the threshold value for decay, $M_{\overline{\sigma}} > 2M_{eff} = 2\sqrt{M_q^2 + 4G^2\pi^2}$. Therefore, there exists no $\overline{\sigma}$ meson in the superfluid, and the coupling constant $g_{\overline{\sigma}q\bar{q}}$ keeps zero at $\mu_I > \mu_I^c$. Due to the formation of pion superfluid, the coupling constants $g_{\sigma q\bar{q}}$ and $g_{\overline{\sigma}q\bar{q}}$ are discontinuous at the critical isospin chemical potential μ_I^c. For the other three mesons $\pi_0, \overline{\pi}_+$ and $\overline{\pi}_-$, their masses are below the threshold value, and the coupling constants are nonzero. From the mass relation $M_{\overline{\pi}_-} \to M_{\pi_0}$ at $\mu_I \to \infty$, the couplings $g_{\overline{\pi}_- q\bar{q}}$ and $g_{\pi_0 q\bar{q}}$ approach to each other at large enough μ_I. Note that $g_{\pi_0 q\bar{q}}$ is no longer a constant in the pion superfluid but changes slowly.

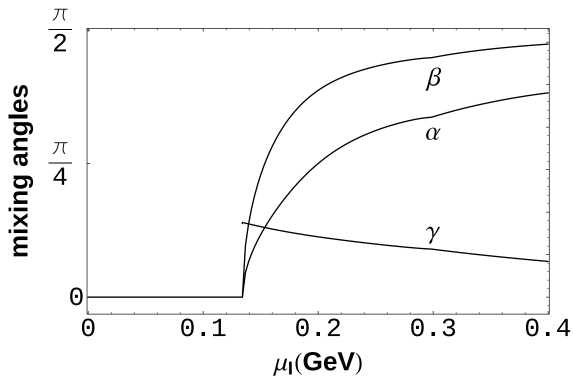

FIG. 1: The mixing angles α, β and γ in the $\overline{\sigma}$-meson channel as functions of μ_I at $T = \mu_B = 0$.

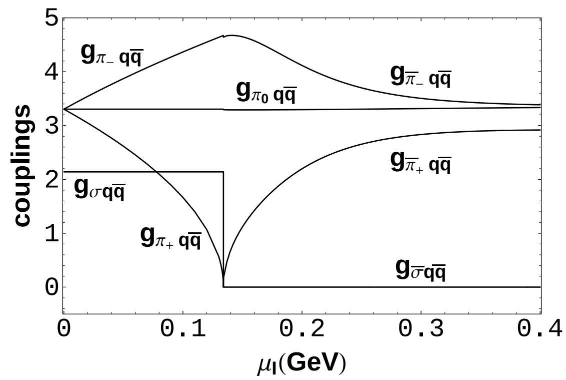

FIG. 2: The coupling constants for $\sigma, \pi_0, \pi_+, \pi_-$ in the normal phase and $\overline{\sigma}, \pi_0, \overline{\pi}_+, \overline{\pi}_-$ in the pion superfluidity phase as functions of μ_I at $T = \mu_B = 0$.

IV. SUMMARY

We have investigated the meson mixing and meson coupling constants in pion superfluid in the NJL model. In the pion superfluidity phase, the normal mesons are no longer the collective excitation modes of the system, and the mixing among them becomes important. For the Goldstone mode, it contains only charged pions and the fractions are exactly the same in the whole superfluid region. For the other new eigen modes, the meson mixing starts to control the system at $\mu_I \gtrsim 150$ MeV which is only a little bit higher than the critical value $\mu_I^c = m_\pi = 134$ MeV. The coupling constants for the conventional mesons in the normal phase and for the new eigen modes in the pion superfluidity phase behavior very differently. These significant changes in the meson properties due to the spontaneous isospin symmetry breaking will help us to understand the dynamic processes, like $\pi\pi$ scattering, happened in neutron stars and heavy ion collisions at intermediate energy.

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